

# Tsai Wu failure criterion applied to carbon fiber reinforced plastics with woven fabric reinforcement

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## Keywords

Tsai-Wu failure criterion, CFRP, water adsorption, DMA, ultrasound evaluation methods

## 1. Introduction

In materials science, material failure is the loss of load carrying capacity of a material unit. This definition introduces the fact that material failure can be examined in different scales, from microscopic, mesoscale to macroscopic. In structural problems, where the structural response should be determined beyond the initiation of nonlinear material behaviour, material failure is of profound importance for the determination of the integrity of the structure. On the other hand, due to the lack of globally accepted fracture criteria, the determination of the structure's damage, due to material failure, is still under intensive research [1].

In the mathematical terms, failure theory is expressed in the form of various failure criteria which are valid for specific materials. Failure criteria are functions in stress or strain space which separate "failed" states from "unfailed" states.

Microscopic material failure is defined in terms of crack propagation and initiation. Such methodologies are useful for gaining insight in the cracking of specimens and simple structures under well-defined global load distributions. Microscopic failure considers the initiation and propagation of a crack.

Macroscopic material failure is defined in terms of load carrying capacity or energy storage capacity, equivalently. Five general levels are considered, at which the meaning of deformation and failure is interpreted differently: the structural element scale, the macroscopic scale where macroscopic stress and strain are defined, the mesoscale which is represented by a typical void, the microscale and the atomic scale. The material behaviour at one level is considered as a collective of its behaviours at a sublevel. An efficient deformation and failure model should be consistent at every level.

The standard fiber composite analyses concern laminates from unidirectional reinforced layers. Many composites components, however, are manufactured using the widely available woven fabric reinforcements, thanks to their good formability characteristics, very often these components are thin walled, for which Kirchhoff's hypothesis for bending of thin plates is valid and hence, for which the classical laminate theory (CLT) is a suitable description [2], [3]. An orthogonal woven fabric reinforced layer can generally be characterized on macroscopic level as an orthotropic material with nine independently elastic engineering constants ( $E_1, E_2, E_3, \nu_1, \nu_2,$

$\nu_3, G_{12}, G_{13}, G_{23}$ ) [4]. For the sake of clarity, the analyses presented here is restricted to fabrics with orthogonal fiber orientation, leaving the effects of skew deformation due to the composite forming process out of consideration. This implies that the fiber directions are normal to planes of material symmetry, and that a shear-extension coupling in the fiber direction will be absent. The fiber directions define the principal material axes, where the directions of the warp yarns is indicated with "1", the direction of the weft or fill yarns with "2", whereas "3" is the direction normal to the plane of the laminate.

## 2. Unidirectional approximation of a woven fabric ply

Six basic ply material properties are necessary for the characterization of woven fabric ply  $\{E_1, E_3, \nu_{12}, \nu_{13}, G_{12}, G_{13}\}^{WF}$ . For the characterization of unidirectional layers are necessary only five elastic parameters  $\{E_1, E_2, \nu_{12}, \nu_{23}, G_{12}\}^{UD}$  because

$$E_3^{UD} = E_2^{UD}; G_{13}^{UD} = G_{12}^{UD}; \nu_{13}^{UD} = \nu_{12}^{UD} \quad (1)$$

Using the method presented in [4], a regular system of five equations is obtained when the transverse shear terms are omitted, leaving

$$\begin{aligned} \left( \frac{2 E_1 [E_1 + (1 - \nu_{12}^2) E_2 - \nu_{12}^2 E_2^2]}{E_1 E_1 (E_1 + 2E_2) + (1 + 2\nu_{12}^2) E_2^2} \right)^{UD} &= \left( \frac{1}{E_1} \right)^{WF} \\ \left( \frac{4 \nu_{12} E_2 (E_1 - \nu_{12}^2 E_2)}{E_1 E_1 (E_1 + 2E_2) + (1 + 2\nu_{12}^2) E_2^2} \right)^{UD} &= \left( \frac{\nu_{12}}{E_1} \right)^{WF} \\ \left( \frac{1 E_1 (\nu_{12} + \nu_{23} + \nu_{12}\nu_{23}) + \nu_{12}^2 E_2}{E_1 + (1 + 2\nu_{12}) E_2} \right)^{UD} &= \left( \frac{\nu_{13}}{E_1} \right)^{WF} \\ \left( \frac{(1 - \nu_{23}^2) E_1^2 + (1 + 2\nu_{12} + 2\nu_{12}\nu_{23}) E_1 E_2 - \nu_{12}^2 E_2^2}{E_1 E_2 [E_1 + (1 + 2\nu_{12}) E_2]} \right)^{UD} &= \left( \frac{1}{E_1} \right)^{WF}; \\ \left( \frac{1}{G_{12}} \right)^{UD} &= \left( \frac{1}{G_{12}} \right)^{WF} \end{aligned} \quad (2)$$

Supplementary, we can use also

$$\frac{1}{G_{13}^{WF}} = \left( \frac{1 + \nu_{23}}{E_2} + \frac{1}{2G_{12}} \right)^{UD} \quad (3)$$

The problem that is set is how much repetitive sequences  $[0^\circ, 90^\circ]$  are necessarily for the simulation of a woven fabric ply. This problem can be solved imposing the condition that the flexural rigidity of the woven fabric ply shall be approximate equal with those of the repetition sequence  $[0^\circ, 90^\circ]$ . According to [5], this might happen if the number of repetitive sequences is at least 6.

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3. Tsai-Wu failure criterion

Tsai-Wu theory is a quadratic failure criterion for anisotropic materials. It is stated as

$$f_i \sigma_i + f_{ij} \sigma_i \sigma_j \leq 1, \quad i, j = 1, 2, \dots, 6 \quad (4)$$

If the failure surface is to be closed and connex, the interaction terms  $F_{ij}$  must satisfy

$$F_{ii} F_{jj} - F_{ij}^2 \geq 0 \quad (5)$$

which implies that all the  $F_{ii}$  terms must be positive.

For plane -stress condition (4) can be write as

$$F_1 \sigma_1 + F_2 \sigma_2 + F_6 \sigma_6 + F_{11} \sigma_1^2 + F_{22} \sigma_2^2 + F_{66} \sigma_6^2 + 2F_{12} \sigma_1 \sigma_2 + 2F_{16} \sigma_1 \sigma_6 + 2F_{26} \sigma_2 \sigma_6 \leq 1 \quad (6)$$

Shear strength is independent of sign of the shear stress, therefore all linear stress terms must vanish. Therefore we get

$$F_1 \sigma_1 + F_2 \sigma_2 + F_{11} \sigma_1^2 + F_{22} \sigma_2^2 + F_{66} \sigma_6^2 + 2F_{12} \sigma_1 \sigma_2 \leq 1 \quad (7)$$

The constants that appear in (7) can be determined as follows: longitudinal tension and compression tests

$$F_1 = \frac{1}{\sigma_{1t}} - \frac{1}{\sigma_{1c}}; \quad F_{11} = \frac{1}{\sigma_{1t} + \sigma_{1c}}$$

transverse tension and compression test -  $F_2 = \frac{1}{\sigma_{2t}} - \frac{1}{\sigma_{2c}}; \quad F_{22} = \frac{1}{\sigma_{2t} + \sigma_{2c}}$ ; shear tests

$F_{66} = \frac{1}{\tau_{12}}$ .  $\sigma_{1t}$ ,  $\sigma_{2t}$  and  $\sigma_{1c}$ ,  $\sigma_{2c}$  are the failure strengths in uniaxial tension and compression, respectively.  $\tau_{12}$  represents the shear strength in plane 1-2.

4. Studied samples

Samples from composite materials, CFRP type with 12 ply biaxial woven fabric 5H satin type have been taken into study. Fiber designation is Tarayca T300 3K. The matrix is from polyphenylenesulphide designation Ticona Fortron. The thickness of the composite is 4.2mm and the volume ratio is 0.5. The composite is made by TenCate Cetex [4]. The

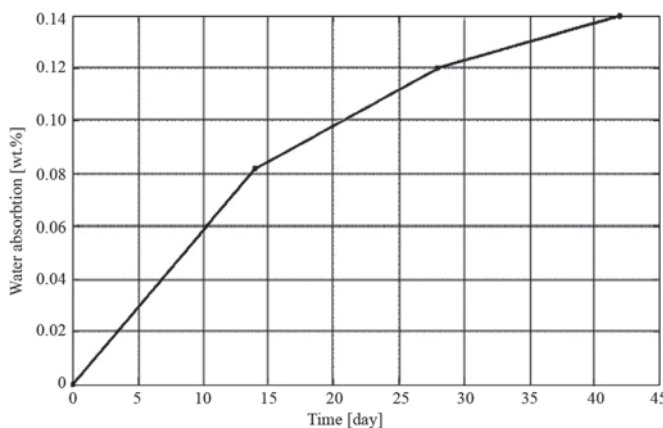


Figure 1. Time dependency of weight adsorbed water percent of the composite at (20±2)°C.

properties of one lamina indicated by the producer are [4]  $\sigma_{1t} = 730\text{MPa}$ ;  $\sigma_{2t} = 646\text{MPa}$ ;  $\sigma_{1c} = 558\text{MPa}$ ;  $\sigma_{2c} = 526\text{MPa}$ ; in plane shear strength = 108MPa;  $E_1 = 57\text{GPa}$ ;  $E_2 = 53\text{GPa}$ ;  $G_{13} = 58\text{GPa}$ ;  $G_{23} = 45\text{GPa}$ ;  $G_{12} = 2.65\text{GPa}$ .

Immersed in water, the composite absorbs a certain quantity of water depending on immersion time. The time dependency of adsorbed water is presented in Figure 1, being determined



Figure 2. Studied samples.

through weighing with analytical balance with 1µg precision. In Figure 2 are presented the studied samples.

5. Experimental set-up

The elastic properties of composites were determined as following:  $E_1$ ,  $E_2$  and  $E_3$  using Dynamic Mechanical Analyzer DMA 242C -Netzsch Germany (Figure 3).

Poisson coefficients  $\nu_{12}$  and  $\nu_{21}$  were obtained by measuring phase velocity of Lamb waves with 65kHz frequency generated

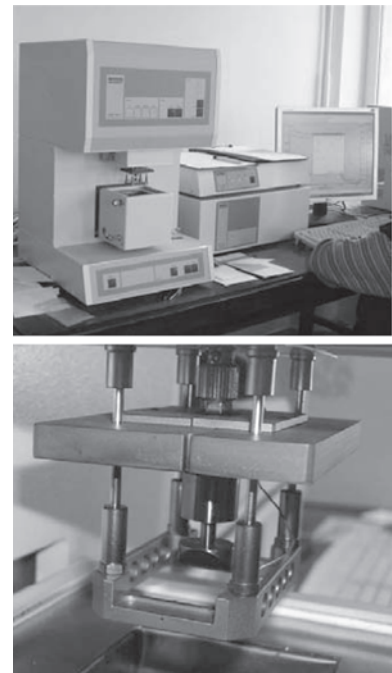


Figure 3. a) Dynamic Mechanical Analyzer DMA 242C; b) 3 points bending test.

in composite by Hertzian contact,  $S_0$  mode - using transducers with Hertzian contact, P111=01-P3.1 Introscope coupled at Pulser Receiver 5077PR Panametrics USA, the digitization being assured by LeCroy Wave Runner 64Xi digital oscilloscope which allows the measuring of time intervals with  $10^{-10}$ s precision Figure 4.

$\nu_{13} = \nu_{23}$  were obtained by measuring the propagation velocity of compressional and shear ultrasonic waves with frequency of 5MHz and 4MHz respectively using US transducers for compressional waves, type G5KB GE and

MB4Y-GE type for transversal waves. The Pulser Receiver was 5073PR type, Panametrics USA and the same oscilloscope.



Figure 4. Ultrasound with Hertzian contact experimental set-up.

Table 1. Water effects on elastic characteristics of studied composite.

Immersion time [days]	0	14	28	42
$E_1$ [GPa]	45	40	34	27
$E_2$ [GPa]	44	29	34	27
$E_3$ [GPa]	11	9	7	6
$\nu_{12}$	0.32	0.31	0.30	0.30
$\nu_{21}$	0.32	0.31	0.30	0.30
$\nu_{13}$	0.03	0.03	0.02	0.02
$G_{12}$ [GPa]	8.4	7.7	6.7	5.9
$G_{21}$ [GPa]	8.4	7.7	6.7	5.9
$G_{13}$ [GPa]	5.5	4.8	4.0	3.1

### 6. Experimental results

The effects of water adsorption over the mechanical properties of the composite are presented in Table 1. These data represent the average value obtained on 3 samples.

For applying Tsai-Wu criterion, a numerical code has been developed in Matlab 2009b, based on finite element method (FEM). The finite element chosen was 4 Node-20 Dof Reissner Flat -Shall [6]. The code allows, in the basis of data from lamina and of efforts at which the CFRP plate was supposed, the calculation of Tsai-Wu number and the direction of global coordinate axis reported to the local ones of the lamina that minimizes the Tsai Wu number. A plate with dimensions  $1000 \times 1000 \times 4.2 \text{ m}^3$  from a composite with the elastic properties indicated above has been analyzed. Strengths and moments that assure Tsai-Wu number almost equal with 1 have been searched. In this way, each of the 12 laminas of the composite, with bidirectional anisotropy, has been fictive decomposed in 8 layers with  $[0^\circ, 90^\circ]$  layout. In CFRP thickness  $12 \times 8 = 96$  elements have been taken, localized on the middle of equivalent laminas. Along x and y directions,  $60 \times 60$  elements have been chosen. The composite plate has been loaded as following  $\sigma_x = 825 \text{ MPa}$ ,  $\nu_y = 700 \text{ MPa}$ ,  $M_x = 885 \text{ MPa} \cdot \text{m}$ ,  $M_y = 700 \text{ MPa} \cdot \text{m}$ . The analyses effectuated with FEM code shows that in these conditions, the minimum of Tsai-Wu number is 0.709 and the optimal angle between the global and local coordinate systems is  $22.5^\circ$ , Figure 5. If the same composite plate is immersed 14 days in water, considering the same loads, it has been ascertained that the minimum value of Tsai-Wu number is 1.874 and this value is obtained for the angle  $76.5^\circ$ , Figure 6.

In this situation, the composite plate failures for the same loads because practically all the elastic parameters are modified due to the action of adsorbed water.

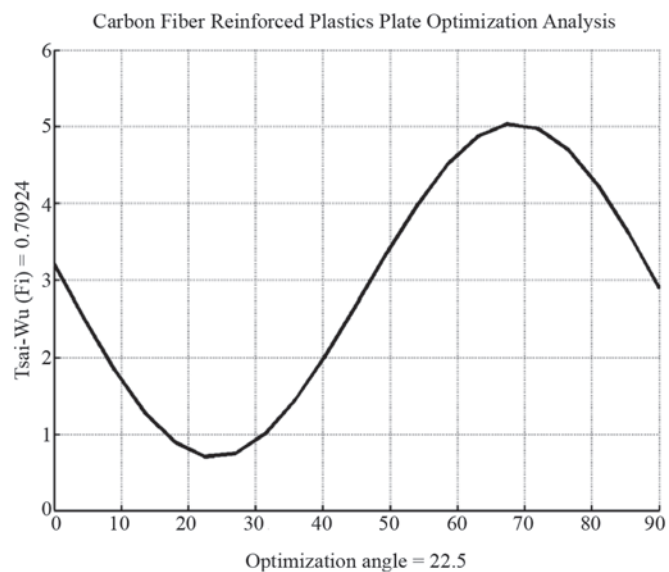


Figure 5. Dependency of Tsai-Wu number by the angle between global and local coordinate systems.

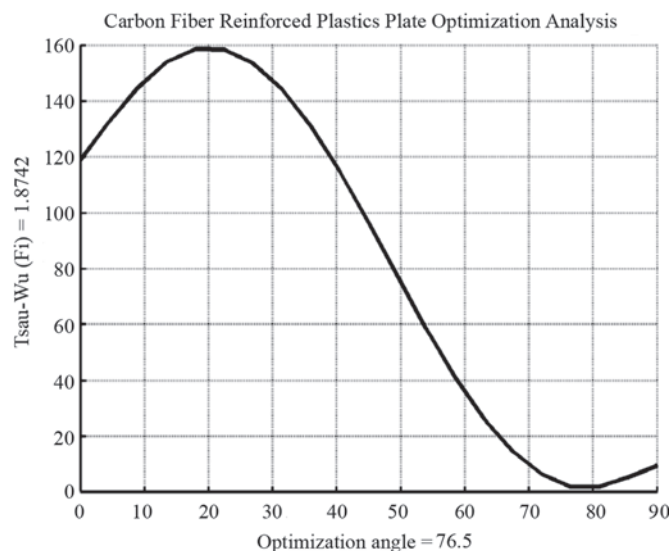


Figure 6. Dependency of Tsai-Wu number by the angle between global and local coordinate systems for the sample with adsorbed water.

### 7. Conclusions

This paper studies the effect of water adsorption by CFRP having reinforcement from woven fabric from carbon fibers 5H satin type and PPS matrix. The elastic properties of the lamina are delivered by the producer and of the composite in assembly have been determined by dynamic mechanical analyzes and ultrasound procedures. Immersing the samples in water, it has been shown that the material adsorb water, the weight percent of the adsorbed water being proportional with the immersion time. The elastic properties of the immersed samples were determined, being shown that the elastic modulus and shear modulus decrease drastically. To use the Tsai Wu failure criterion, a method of transformation of the laminas with bidirectional anisotropy in  $[0^\circ, 90^\circ]$  fictive sequences with the same properties was presented. A system of loading was chosen so that the reduced angular interval

between global and local coordinate system attached to the plies assure a Tsai -Wu number smaller than 1. The composite immersed 14 days in water, at the same loads, failures, the Tsai-Wu number having relative higher values.

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